Hong Kong Mathematics Olympiad (2000 – 2001)

Final Event 1 (Individual)

香港数学竞赛 **(2000 – 2001)**

决赛项目 **1 (个人)**

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. a 、 b 和 c 分别为 ΔABC 的 $\angle A$、$\angle B$ 和 $\angle C$ 的相对边的长度。若 $\angle C = 60°$ 及 $\frac{a}{b+c}+\frac{b}{a+c}=P$，求 P 的值。

   a , b and c are the lengths of the opposite sides of $\angle A$, $\angle B$ and $\angle C$ of the ΔABC respectively . If $\angle C = 60°$ and $\frac{a}{b+c}+\frac{b}{a+c}=P$, find the value of P .

2. 已知 $f(x)=x^2+ax+b$ 是 x^3+4x^2+5x+6 和 $2x^3+7x^2+9x+10$ 的公因式。若 $f(P)=Q$，求 Q 的值。

   Given that $f(x)=x^2+ax+b$ is a common factor of x^3+4x^2+5x+6 and $2x^3+7x^2+9x+10$. If $f(P)=Q$, find the value of Q .

3. 已知 $\frac{1}{a}+\frac{1}{b}=\frac{Q}{a+b}$ 及 $\frac{a}{b}+\frac{b}{a}=R$，求 R 的值。

   Given that $\frac{1}{a}+\frac{1}{b}=\frac{Q}{a+b}$ and $\frac{a}{b}+\frac{b}{a}=R$, find the value of R .

4. 已知 $\begin{cases} a+b=R \\ a^2+b^2=12 \end{cases}$ 及 $a^3+b^3=S$，求 S 的值。

   Given that $\begin{cases} a+b=R \\ a^2+b^2=12 \end{cases}$ and $a^3+b^3=S$, find the value of S .

Hong Kong Mathematics Olympiad (2000 – 2001)

Final Event 2 (Individual)

香港数学竞赛 (2000 – 2001)

决赛项目 2 (个人)

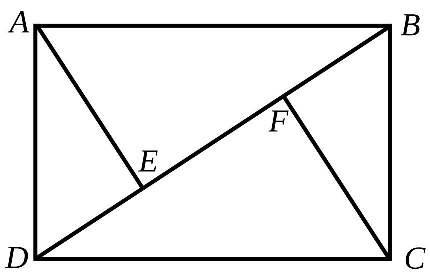
除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若 P 为整数，及 $5 < P < 20$。若方程 $x^2 - 2(2P-3)x + 4P^2 - 14P + 8 = 0$ 的两个根皆为整数，求 P 的值。

Suppose P is an integer and $5 < P < 20$. If the roots of the equation $x^2 - 2(2P-3)x + 4P^2 - 14P + 8 = 0$ are integers, find the value of P.

2. $ABCD$ 是一长方形。若 $AB = 3P + 4$， $AD = 2P + 6$，AE 和 CF 分别垂直于对角线 BD，及 $EF = Q$，求 Q 的值。

$ABCD$ is a rectangle. $AB = 3P + 4$, $AD = 2P + 6$. AE and CF are perpendiculars to the diagonal BD. If $EF = Q$, find the value of Q.



3. 某班学生的人数少于 $4Q$ 人。在一次数学测验中有 $\frac{1}{3}$ 学生得甲等，$\frac{1}{7}$ 学生得乙等，一半学生得丙等，余下的学生都不合格。已知不合格的学生人数是 R，求 R 的值。

There are less than $4Q$ students in a class. In a mathematics test, $\frac{1}{3}$ of the students got grade A, $\frac{1}{7}$ of the students got grade B, half of the students got grade C, and the rest failed. Given that R students failed in the mathematics test, find the value of R.

4. $[a]$ 表示不大于 a 的最大整数。例如 $[2\frac{1}{3}]=2$。已知方程 $[3x+R]=2x+\frac{3}{2}$ 的所有根的和为 S，求 S 的值。

$[a]$ represents the largest integer not greater than a. For example, $[2\frac{1}{3}]=2$. Given that the sum of the roots of the equation $[3x+R]=2x+\frac{3}{2}$ is S, find the value of S.

Hong Kong Mathematics Olympiad (2000 – 2001)

Final Event 3 (Individual)

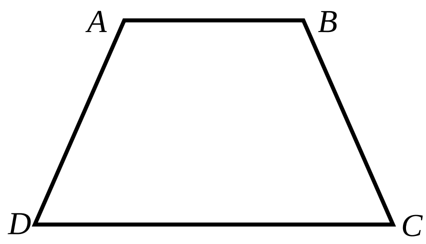
香港数学竞赛 **(2000 – 2001)**

决赛项目 **3 (个人)**

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. $ABCD$ 是一梯形，其 $\angle ADC = \angle BCD = 60^\circ$ 及 $AB = BC = AD = \frac{1}{2}CD$。若把这梯形分割为 P 等分 ($P > 1$)，使其分割所得的每份与梯形 $ABCD$ 相似。求 P 的最小值。

   $ABCD$ is a trapezium such that $\angle ADC = \angle BCD = 60^\circ$ and $AB = BC = AD = \frac{1}{2}CD$. If this trapezium is divided into P equal portions ($P > 1$) and each portion is similar to trapezium $ABCD$ itself. Find the minimum value of P.



2. $(P+1)^{2001}$ 的个位数字与十位数字之和是 Q，求 Q 的值。

   The sum of the tens and unit digits of $(P+1)^{2001}$ is Q. Find the value of Q.

3. 若 $\sin 30^\circ + \sin^2 30^\circ + \cdots + \sin^Q 30^\circ = 1 - \cos^R 45^\circ$，求 R 的值。

   If $\sin 30^\circ + \sin^2 30^\circ + \cdots + \sin^Q 30^\circ = 1 - \cos^R 45^\circ$, find the value of R.

4. 设方程 $x^2-8x+(R+1)=0$ 的根为 α 和 β。若 $\frac{1}{\alpha^2}$ 和 $\frac{1}{\beta^2}$ 是方程 $225x^2-Sx+1=0$ 的根，求 S 的值。

Let α and β be the roots of the equation $x^2-8x+(R+1)=0$. If $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ are the roots of the equation $225x^2-Sx+1=0$, find the value of S.

Hong Kong Mathematics Olympiad (2000 – 2001)

Final Event 4 (Individual)

香港数学竞赛 **(2000 – 2001)**

决赛项目 **4 (**个人**)**

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知 $a^{\frac{2}{3}}+b^{\frac{2}{3}}=17\frac{1}{2}$， $x=a+3a^{\frac{1}{3}}b^{\frac{2}{3}}$， $y=b+3a^{\frac{2}{3}}b^{\frac{1}{3}}$。若 $P=(x+y)^{\frac{2}{3}}+(x-y)^{\frac{2}{3}}$，求 P 的值。

Let $a^{\frac{2}{3}}+b^{\frac{2}{3}}=17\frac{1}{2}$, $x=a+3a^{\frac{1}{3}}b^{\frac{2}{3}}$ and $y=b+3a^{\frac{2}{3}}b^{\frac{1}{3}}$. If $P=(x+y)^{\frac{2}{3}}+(x-y)^{\frac{2}{3}}$, find the value of P .

2. 若一正 Q 边形有 P 条对角线，求 Q 的值。

If a regular Q-sided polygon has P diagonals , find the value of Q .

3. 已知 $x=\sqrt{\frac{Q}{2}+\sqrt{\frac{Q}{2}}}$， $y=\sqrt{\frac{Q}{2}-\sqrt{\frac{Q}{2}}}$。若 $R=\frac{x^6+y^6}{40}$，求 R 的值。

Let $x=\sqrt{\frac{Q}{2}+\sqrt{\frac{Q}{2}}}$ and $y=\sqrt{\frac{Q}{2}-\sqrt{\frac{Q}{2}}}$. If $R=\frac{x^6+y^6}{40}$, find the value of R .

4. 已知 $[a]$ 表示不大于 a 的最大整数。例如 $[2.5]=2$。若 $S=\left[\frac{2001}{R}\right]+\left[\frac{2001}{R^2}\right]+\left[\frac{2001}{R^3}\right]+\cdots$，求 S 的值。

$[a]$ represents the largest integer not greater than a . For example , $[2.5]=2$. If $S=\left[\frac{2001}{R}\right]+\left[\frac{2001}{R^2}\right]+\left[\frac{2001}{R^3}\right]+\cdots$, find the value of S .